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III клас Диференциален бином

$$\int x^m (a + bx^n)^p dx$$

m, n, p - рационални числа
но не са фиксирани

I в p - цяло; II в $\frac{m+1}{n}$ - цяло, III в $\frac{m+1}{n} + p$ - цяло

следната субституция

при p : $x = t^k$ k - НОК на знаменателя $\{m, n\}$
при $\frac{m+1}{n}$: $z = x^n$
при $\frac{m+1}{n} + p$: $z = x^n$

Разглеждаме I-ви случай

$$x = t^k \Rightarrow t = x^{\frac{1}{k}} \quad dx = k t^{k-1} dt$$

$$x^m = t^{km} \Rightarrow \int t^{km} (a + bt^{kn})^p k t^{k-1} dt$$

получаваме табличен полином

II случай

$$z = x^n \Rightarrow x = z^{\frac{1}{n}} \Rightarrow dx = \frac{1}{n} z^{\frac{1}{n}-1} dz$$

$$\frac{1}{n} \int z^{\frac{m+1}{n} - 1} (a + bz) dz$$ от $dx = p$, p-го представяне

от вида $p = \frac{r}{e} \Rightarrow$

$$\frac{1}{n} \int z^{\frac{m+1}{n} - 1} \left(\sqrt[e]{(a + bz)} \right)^e dz$$

полагане

$$\sqrt[e]{(a+bz)} = t \Rightarrow a + bz = t^e \Rightarrow bz = t^e - a$$

$$z = \frac{1}{b} t^e - \frac{a}{b}, \quad dz = \frac{e}{b} t^{e-1} dt$$

$$\frac{e}{bn}\int\left(\frac{1}{e}t^e-\frac{a}{e}\right)^{\overbrace{\frac{m+1}{n}-1}^{\text{цяло}}}t^{\overbrace{r+e-1}^{\text{цяло}}}dt$$

$-$ цяло

$$\frac{m+1}{n}+p$$

от $z=x^n$ вземаме и заместваме

$$\frac{1}{n}\int z^{\frac{m+1}{n}-1}(a+bz)^p\,dz$$

делим и умножаваме на степен p т.е.

n-го представяне $p=\frac{e}{e}$

$$\frac{1}{n}\int z^{\overbrace{p+\frac{m+1}{n}-1}^{\text{цяло}}}\left(\frac{e\sqrt{a+bz}}{z}\right)^z dz$$

$$\int\frac{dx}{x\sqrt{1+\sqrt{x}}}\quad;\quad\int\frac{dx}{\sqrt{x}\sqrt[4]{1+x^2}}$$

$n=1$ от $x^1\sqrt{1+\sqrt{x}}$, $p=\frac{1}{2}$

$\frac{m+1}{n}$ – цяло $\Rightarrow$ полагаме $z=x^n$ т.е.

$$z=x^{\frac{1}{2}}\Rightarrow x=z^2$$

$dx=2z$ заместваме в интеграла

$$\int\frac{1}{z^2\sqrt{1+z}}\cdot 2z\,dz$$ – полагаме $(1+z)^{1/2}=t\Rightarrow$

$$1+z=t^2\Rightarrow z=t^2-1$$

$$dz=(t^2-1)'dt=2t\,dt\Rightarrow$$

$$\int \frac{1}{(t^2-1)\sqrt{t^2}} 2 \cdot 2t\,dt = \int \frac{1}{(t^2-1)t} 4t\,dt =$$

$$4\int \frac{1}{(t-1)(t+1)}\,dt \Rightarrow \frac{1}{(t-1)(t+1)} = \frac{A}{(t-1)} + \frac{B}{(t+1)} =$$

$$A(t+1) + B(t-1) = 1 \Rightarrow t = 1,\ t = -1 \text{ т. е.}$$

$$\left.\begin{array}{l} 2A = 1 \Rightarrow A = \frac{1}{2} \\ -2B = 1 \Rightarrow B = -\frac{1}{2} \end{array}\right\} \Rightarrow 4\int \frac{1}{2(t+1)} - \frac{1}{2(t-1)}\,dt =$$

$$2\int \left(\frac{1}{t+1} - \frac{1}{t-1}\right) dt = 2\left(\ln|t+1| - \ln|t-1|\right) =$$

$$2\ln\left|\frac{t+1}{t-1}\right|$$

връщаме до $x \Rightarrow 2\ln\left|\frac{\sqrt{1+z}+1}{\sqrt{1+z}-1}\right| =$

$$2\ln\left|\frac{\sqrt{1+\sqrt{x}}+1}{\sqrt{1+\sqrt{x}}-1}\right|$$

$$\int \frac{dx}{\sqrt{x}\sqrt{1+x^2}} \qquad m = 1/2 \qquad n = 2$$

$$p = 1/4 \Rightarrow \text{Чебишов}$$

полагаме $x^n = z$, $z = x^2 \Rightarrow x = z^{1/2}$

$$dx = \frac{1}{2} z^{1/2-1}\,dz$$

$$\int \frac{1}{\sqrt{z^{\frac{1}{2}}}\sqrt[4]{1+z^{\frac{1}{2}}}^2}\ \frac{1}{2}\ z^{1/2}\,dz = \frac{1}{2}\int \frac{1}{\sqrt[4]{z}\sqrt[4]{1+z}}\ z^{1/2}\,dz =$$

$$\frac{1}{2}\int \frac{1}{\sqrt[4]{z(1+z)}}\ z^{1/2}\,dz = \frac{1}{2}\int \frac{1}{\sqrt[4]{z(1+z)}}\cdot\frac{1}{\sqrt{z}}\,dz =$$

делим и умножаем $\cos z$

$$\frac{1}{2}\int \frac{1}{z\sqrt[4]{\frac{1+z}{z}}}\,dz \quad \text{полагаем } \sqrt[4]{\frac{1+z}{z}} = t \Rightarrow \frac{1+z}{z} = t^4$$

$$z = t^4 - 1 \Rightarrow z(1-t^4) = -1$$

$$z = -\frac{1}{1-t^4} = \frac{1}{t^4-1} \Rightarrow z = (t^4-1)^{-1}$$

$$dz = -1(t^4-1)^{-2}\,4t^3\,dt$$

$$\frac{1}{2}\int \frac{t^4-1}{t}\left(-(t^4-1)^{-2}\,4t^3\right)dt$$

$$\frac{1}{2}\int \frac{t^4-1}{t}\cdot\frac{-4t^3}{(t^4-1)^2}\,dt = -\frac{1}{2}\cdot 4\int \frac{t^2}{t^4-1}\,dt = -2\int \frac{t^2}{t^4-1}$$

$$\frac{t^2}{t^4-1} = \frac{t^2}{(t^2+1)(t^2-1)} = \frac{t^2}{(t^2+1)(t+1)(t-1)} =$$

$$A\ln|t-1| + B\ln|t+1| + \frac{N}{2}\ln|1+t^2| + N\operatorname{arctg} t$$